New Massive Gravity: beyond 3D

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Work in progress

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Outline

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Why Higher-Curvature Gravity ?

Consider 4D Einstein gravity as a theory of interacting massless spin 2 particles around a Minkowski space-time background

Problem: This theory is perturbative non-renormalizable

$$\mathcal{L} \sim R + a\left(R_{\mu\nu}{}^{ab}\right)^2 + b\,(R_{\mu\nu})^2 + c\,R^2\ :$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no massless spin 2 !

$\Rightarrow$ "New Massive Gravity"

- Can this be extended to higher dimensions ?

D=4 : phenomenological applications ?

for a review, see Hinterbichler (2011)

Outline

Massless Degrees of Freedom

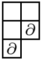
field $S \sim \ydiagram{2,1}$

gauge parameters $\lambda_1 \sim \ydiagram{1,1}$ $\lambda_2 \sim \ydiagram{2}$

gauge transformation $\delta\, \ydiagram{2,1} = \ytableaushort{{}\partial,{}} + \ytableaushort{{}{},\partial}$

curvature $R(S) \sim \ytableaushort{{}{},{}\partial,\partial}$

Zero Massless D.O.F.

"Einstein tensor" $G(S) \sim \star\star$ 

Requirement : $G(S) \sim$  $\Rightarrow$ E.O.M. : $G(S) = 0$

$$s = 2 \; : p + q = D - 1$$

Example : $p = q = 1\,, D = 3\,,$ $S \sim$ 

"Boosting Up the Derivatives"

Massive Generalized FP

Curtright (1980)

$$\left(\Box - m^2\right)\, S = 0\,, \qquad S^{\rm tr} = 0\,, \quad \partial \cdot S = 0$$

$$\partial \cdot S = 0 \quad \Rightarrow \quad S = G(T)$$

$$\left(\Box - m^2\right)\, G(T) = 0\,, \qquad G(T)^{\rm tr} = 0$$

Higher-derivative Gauge Theory

Outline

3D Einstein-Hilbert Gravity

There are no massless gravitons

Adding higher-derivative terms leads to “massive gravitons”

Fierz-Pauli

- $\left(\Box - m^2\right) h_{\mu\nu} = 0\,, \qquad h_{\mu\nu} = h_{\nu\mu}\,, \quad \eta^{\mu\nu} h_{\mu\nu} = 0\,, \quad \partial^\mu h_{\mu\nu} = 0$

- $\mathcal{L}_{\rm FP} = \frac{1}{2} h^{\mu\nu} G_{\mu\nu}(h) + \frac{1}{2} m^2 \left(h^{\mu\nu} h_{\mu\nu} - h^2\right)\,, \qquad h \equiv \eta^{\mu\nu} h_{\mu\nu}$

no non-linear extension !

number of propagating modes is $\frac{1}{2} D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Note: the numbers become 2 (4D) and 0 (3D) for $m = 0$

Higher-derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \epsilon_\nu{}^{\gamma\delta} \partial_\alpha \partial_\gamma h_{\beta\delta} \equiv G_{\mu\nu}(h)$$

$$\left(\Box - m^2\right) \, G^{\text{lin}}_{\mu\nu}(h) = 0 \,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g} \left[-R + \frac{1}{m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right) \right]$$

"New Massive Gravity" : unitary !

Mode Analysis

- Take NMG with metric $g_{\mu\nu}$, cosmological constant Λ and coefficient $\sigma = \pm 1$ in front of R
- lower number of derivatives from 4 to 2 by introducing an auxiliary field $f_{\mu\nu}$
- after linearization and diagonalization the two fields describe a massless spin 2 with coefficient $\bar{\sigma} = \sigma - \frac{\Lambda}{2m^2}$ and a massive spin 2 with mass $M^2 = -m^2\bar{\sigma}$
- special cases: 3D NMG and $D \geq 3$ "critical gravity" for special value of Λ

What did we learn?

- two theories can be equivalent at the linearized level (FP and boosted FP) but only one of them allows for a non-linear extension i.e. interactions !

- we need massive spin 2 whose massless limit describes 0 d.o.f.

Example : $\square\!\square$ in 3D

- what about 4D?

Outline

Introduction

General Spin

3D “New Massive Gravity”

“New Massive” 4D Gravity

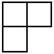
Conclusions

Generalized spin-2 FP

standard spin-2 :

describes $\begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 2 \quad \text{d.o.f.} \quad m = 0 \end{cases}$

generalized spin-2 :



describes $\begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 0 \quad \text{d.o.f.} \quad m = 0 \end{cases}$

Curtright (1980)

Connection-metric Duality

- start with first-order form of EH with independent fields ${e_\mu}^a$ and ${\omega_\mu}^{ab}$

- linearize around Minkowski: ${e_\mu}^a = {\delta_\mu}^a + {h_\mu}^a$

  and add a FP mass term $-m^2(h^{\mu\nu}h_{\nu\mu} - h^2) \quad \rightarrow$

$$\mathcal{L} \;\sim\; h\,\partial\omega + \omega^2 + m^2\,h^2$$

- solve for $\omega \;\rightarrow$ spin-2 FP in terms of h and auxiliary $h_{[\mu\nu]}$

- solve for $h_{\mu\nu}$ and write ${\omega_\mu}^{ab} = \frac{1}{2}\epsilon^{abcd}\,S_{\mu cd} \;\rightarrow$ generalized spin-2 FP in terms of S after elimination of auxiliary $S_{[\mu cd]}$

"New Massive" 4D Gravity

- start with generalized spin-2 FP in terms of

and subsidiary conditions

$$S_{\mu\nu,\rho}\,\eta^{\nu\rho} = 0\,, \qquad \partial^\rho\, S_{\rho\mu,\nu} = 0$$

- solve for $\partial^\rho\, S_{\rho\mu,\nu} = 0 \;\rightarrow\; S_{\mu\nu,\rho} = G_{\mu\nu,\rho}(T) \;\rightarrow$

$$\mathcal{L}_{\mathrm{NMG}} \;\sim\; -\tfrac{1}{2}\,T^{\mu\nu,\rho}\,G_{\mu\nu,\rho}(T) \;+\; \frac{1}{2m^2}\;\underbrace{T^{\mu\nu,\rho}\;{\color{red}C_{\mu\nu,\rho}(T)}}_{\text{"conformal invariance"}}$$

- mode analysis $\rightarrow$

$\mathcal{L}_{\mathrm{NMG}} \;\sim$ massless spin 2 plus massive spin 2

Outline

- dimensional reduction
- massless limit (van Dam-Veltman discontinuity)
- interactions ?
- compare to Einstein-Schrödinger theory

$$\mathcal{L}_{\text{ES}} = \sqrt{-\det R_{(\mu\nu)}(\Gamma)} \quad \Leftrightarrow \quad \mathcal{L}'_{\text{ES}} = \sqrt{-\det g}\,[g^{\mu\nu} R_{\mu\nu}(\Gamma) - \Lambda]$$

- generalization to higher spin ?